ISSN 0012-2661, Differential Equations, 2018, Vol. 54, No. 7, pp. 881–888. © Pleiades Publishing, Ltd., 2018.
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NUMERICAL METHODS

Generalized Collocation Method for Integro-Differential Equations in an Exceptional Case

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Received December 29, 2017

Abstract—We study a linear integro-differential equation with a coefficient that has finite-order zeros. To solve the equation approximately in a distribution space, we suggest and substantiate a generalized collocation method based on special interpolation polynomials.

DOI: 10.1134/S0012266118070054

The object of study in this paper is the linear integro-differential equation of the third kind

$$Ax \equiv x(t)\prod_{j=1}^{q}(t-t_j)^{m_j} + \sum_{j=0}^{p}\int_{-1}^{1} K_j(t,s)x^{(j)}(s)\,ds = y(t), \tag{1}$$

where $t \in I \equiv [-1,1]$, $t_j \in (-1,1)$, $m_j \in \mathbb{N}$ $(j = 1,\dots,q)$, the K_j $(j = 0,\dots,p)$ and y are given "smooth" functions, and x is the unknown function. The study of such equations is of interest both theoretically (in particular, Eq. (1) is a generalization of several classes of linear Fredholm type integral equations) and from the applied viewpoint. Such equations arise in a number of important problems in the theories of neutron transport, elasticity, particle scattering (e.g., see [1; 2, pp. 121–129 of the Russian translation] and the references in [1]), differential equations of mixed type [3], and some loaded integro-differential equations [4]. The integro-differential equations of the third kind considered here can only be solved exactly in very rare special cases, and hence it is especially important to develop and theoretically justify efficient approximate methods for these equations. Several results in this direction were obtained in [5–9], where special direct methods for solving Eq. (1) in a V-type space of distributions generated by the Hadamard finite part of the integral were proposed and justified. Some polynomial and spline methods for solving Eq. (1) in a D-type space of distributions based on the Dirac delta function were developed in [10–12].

In the present paper, we propose a generalized version of the collocation method specially designed for the approximate solution of Eq. (1) in a D-type space. Attention is mainly paid to the justification of this method in the sense of the monograph [13, Ch. 1]. Namely, we prove the existence and uniqueness theorem for the solution of the corresponding approximate equation, estimate the error of the approximate solution, and prove the convergence of the sequence of approximate solutions to the exact solution in the distribution space. We also study the stability and conditionality of the approximate equations.

1. TEST FUNCTION SPACES

Let $C \equiv C(I)$ be the Banach space of continuous functions on the interval I with the usual max norm, and let $m \in \mathbb{N}$. Following [14], we say that a function $f \in C$ belongs to the class